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Self-affine roughness effects on the contact area between elastic bodies

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We have calculated the real contact area between elastic bodies with self-affine rough surfaces, which are described in terms of analytical correlation models in Fourier space. It is found that the roughness has a strong influence on the real contact area $A(\lambda)$ at lateral length scales λ which are comparable with the in-plane roughness correlation length ξ , and for significant applied loads σ_o beyond the linear regime (or $A \propto \sigma_o$). The effect of the roughness exponent H can be rather complex, depending on the relative magnitude of the roughness correlation length ξ with respect to the lateral length scale λ where the contact area is considered. Finally, we also show that descriptions of the influence of the roughness that is only based on power law approximations of the self-affine roughness spectrum are rather insufficient, especially for large roughness exponents H (>0.5).

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I. INTRODUCTION

A diverse variety of surfaces encountered in technological applications possess roughness over a wide range of lateral length scales that span the regime from hundreds of micrometers down to the subnanometer range. Therefore, when two solid bodies, which are macroscopically flat are brought into contact, the real contact area will be only a fraction (depending on the characteristic roughness length scales) of the apparent macroscopic area. Indeed, the real contact area can be thought as the area composed of asperities of one solid body, which are squeezed against asperities of the other body. These asperities can deform elastically or plastically depending of the loading conditions and the material involved in the contact process.

The determination of the real contact area is a fundamental problem with important technological implications.^{1–5} The latter include, for example, heat transfer phenomena between solid bodies, electrical transport, sliding friction,⁶ adhesive forces between solid bodies in direct contact⁴ etc. Furthermore, the real contact area can vary nonlinearly with the loading force that pushes together two solid bodies. Hertz⁷ proved that the contact area of two elastic bodies of quadratic profile varies nonlinearly with the loading force F , namely $\propto F^{2/3}$. However, for a fractal rough surface where small spherical humps are distributed on top of larger ones,⁸ it was found that the real contact area varies linearly with applied load F . A similar conclusion was drawn from other studies where the roughness was approximated with asperities with spherical summits and a Gaussian height distribution.^{1,3} Approximation of the asperity summits with paraboloids yielded also a real contact area, which varies again linearly with the applied load as long as the latter remains low.²

The case of random self-affine rough surfaces, which is known to occur over a wide variety of surfaces and interfaces,^{9,10} has been also investigated in the past in contact phenomena.^{4,5} However, these studies described calculations of the real contact area for weak applied loads ($\ll E$; with E the elastic modulus), which, in addition, were based on extrapolations between asymptotic limits and scaling relations of the self-affine roughness spectrum in Fourier space. Under these conditions, it was shown that the real contact area $A(\lambda)$ at lateral length scale λ varies as a power law $A(\lambda) \propto \lambda^{1-H}$.⁵ The parameter H is the roughness exponent that characterizes the degree of surface irregularity at short lateral length scales ($<\xi$; with ξ in-plane roughness correlation length).

Nevertheless, these studies did not consider a systematic calculation of self-affine roughness effects in combination with the case of high applied loads. This will be the topic of our study where calculations of the real contact area will be performed in terms of self-affine roughness models, which describe the whole range of roughness exponents $0 \leq H \leq 1$ [from logarithmic ($H=0$) to closely Gaussian roughness correlation ($H=1$)]. Moreover, we shall show that the dependence of the real contact area $A(\lambda)$ on the roughness exponent H is more complicated than the previous predictions, even for weak applied loads.⁵ Finally, comparisons with former studies will also be performed.⁵

II. CONTACT THEORY BETWEEN ROUGH SURFACES

Under conditions of frictionless contact between two elastic solids with rough surfaces, the contact stresses depend only on the shape of the gap between the solids prior to any loading.^{4,5} The actual system can be described by a flat elastic surface of Poisson's ratio ν and elastic modulus E , which is in contact with a rigid body that possesses a surface roughness profile reproducing the same undeformed gap between the surfaces.^{4,5} The parameters E and ν are related to the corresponding parameters of the two elastic solids via the relation $(1 - \nu^2)/E = \sum_{k=1,2} (1 - \nu_k^2)/E_k$. If L is on the order

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of the diameter of the macroscopic contact area, the stress distribution $P(\sigma, Y)$ in the contact area under magnification $Y=L/\lambda$ is given by the differential equation^{4,5}

$$\frac{\partial P}{\partial Y} = \left(\frac{dG}{dY} \right) \sigma_o^2 \frac{\partial^2 P}{\partial \sigma^2}$$

with

$$G(Y) = \frac{1}{8} \left[\frac{E}{(1-\nu^2)\sigma_o} \right]^2 \int_{2\pi/L < q < 2\pi Y/L} q^2 C(q) dq, \quad (1)$$

where $C(q)$ is the Fourier transform (roughness spectrum) of the height–height auto correlation function $C(r) = \langle h(\mathbf{r})h(0) \rangle$ with $h(\mathbf{r})$ the surface roughness height such that $\langle h \rangle = 0$. $\langle \dots \rangle$ stands as an ensemble average over possible roughness configurations.

Assuming only elastic deformation (infinite yield stress), we can obtain the ratio $P(Y=L/\lambda)$ of the real contact area $A(\lambda)$ at lateral length scale λ (if the surface was smooth on all length scales shorter than λ ; or apparent area of contact on the length scale λ) to that of the macroscopic contact area $A(L)$,^{4,5} by the equation $P(Y) = \int_0^{+\infty} P(\sigma, Y) d\sigma$. The solution of Eq. (1) with the boundary conditions $P(\sigma=0, Y)=0$ (absence of adhesion) and $P(\sigma=+\infty, Y)=\infty$ yields for $P(Y)$ ^{4,5}

$$P(Y) = \frac{2}{\pi} \int_0^{+\infty} \frac{\sin x}{x} e^{-x^2 G(Y)} dx = \text{erf} \left(\frac{1}{2\sqrt{G(Y)}} \right), \quad (2)$$

which is the main parameter under investigation in the following sections. Calculation of $P(Y)$ requires the knowledge of $G(Y)$ and thus of the roughness spectrum $C(q)$.

III. SELF-AFFINE SURFACE ROUGHNESS MODEL

A wide variety of surfaces/interfaces are well described by a kind of roughness associated with self-affine fractal scaling.^{9,10} For self-affine surface roughness $C(q)$ scales as a power-law $C(q) \propto q^{-2-2H}$ if $q\xi \gg 1$, and $C(q) \propto \text{const}$ if $q\xi \ll 1$.^{9,10} The roughness exponent H is a measure of the degree of surface irregularity,^{9,10} such that small values of H

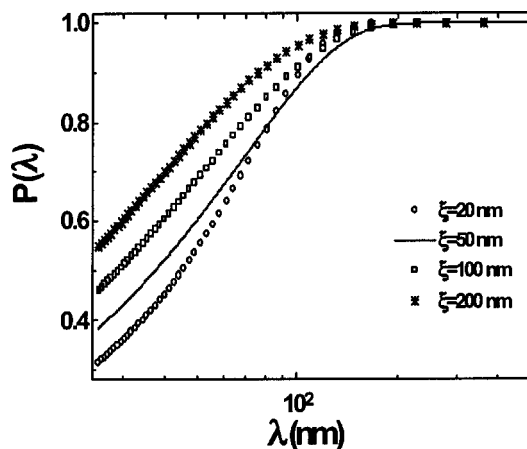


FIG. 1. $P(\lambda)$ vs lateral length scale λ for roughness exponent $H=0.3$, relatively high applied stress σ_o such that $E/\sigma_o=5$, and various correlation lengths ξ .

characterize more jagged or irregular surfaces at short length scales ($< \xi$). This scaling behavior is satisfied by the simple Lorentzian form¹¹

$$C(q) = \frac{1}{2\pi} \frac{w^2 \xi^2}{(1 + a q^2 \xi^2)^{1+H}} \quad (3)$$

with $a = (1/2H)[1 - (1 + a Q_c^2 \xi^2)^{-H}]$ if $0 < H < 1$ (power-law roughness), and $a = (1/2) \ln[1 + a Q_c^2 \xi^2]$ if $H=0$ (logarithmic roughness).¹¹ Moreover, we have $Q_c = \pi/a_o$ with a_o on the order of atomic dimensions, while the parameter w is the rms roughness amplitude. For other correlation models see also Refs. 10 and 12.

IV. RESULTS AND DISCUSSION

Substitution of Eq. (3) into Eq. (1) for the factor $G(Y)$ yields the simple analytic expression

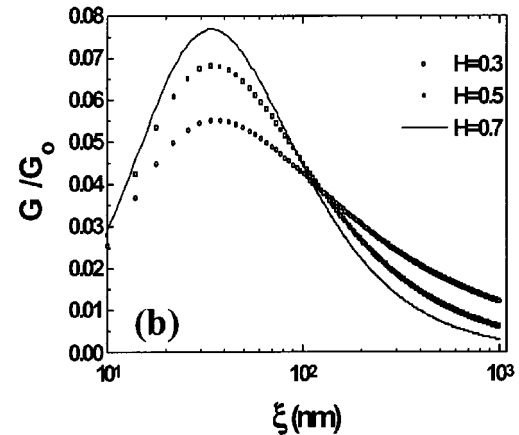
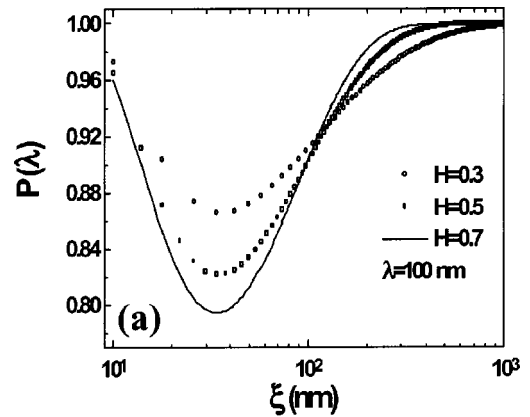


FIG. 2. (a) $P(\lambda)$ vs roughness correlation length ξ for lateral length scale $\lambda=100$ nm, various roughness exponents H , and relatively high applied stress σ_o such that $E/\sigma_o=5$. (b) Calculations of the roughness factor G vs roughness correlation length ξ for various roughness exponents H , $G_o = (1/8)[E/(1-\nu^2)\sigma_o]^2$, $E/\sigma_o=5$, and $\lambda=100$ nm.

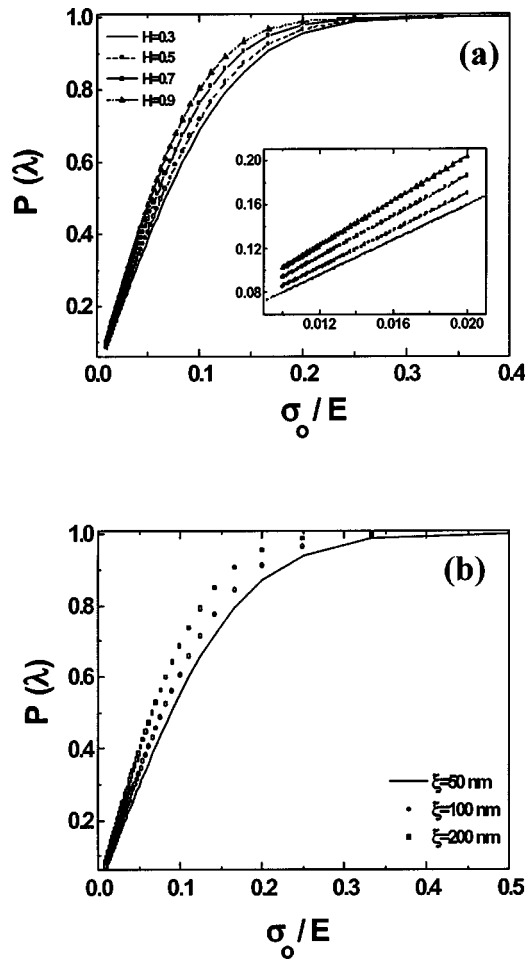


FIG. 3. (a) $P(\lambda)$ vs applied load σ_o/E for roughness correlation length $\xi = 200 \text{ nm}$, lateral length scale $\lambda = 100 \text{ nm}$, and various roughness exponents H . The inset shows the linear regime for small loads ($\sigma_o/E \ll 1$). (b) Similar plot for various correlation lengths and roughness exponent $H = 0.3$.

$$G(Y) = \frac{1}{16} \left[\frac{E}{(1-\nu^2)\sigma_o} \right]^2 \frac{w^2}{a^2 \xi^2} \times \left[\frac{1}{1-H} \{T_{YL}^{1-H} - T_L^{1-H}\} + \frac{1}{H} \{T_{YL}^{-H} - T_L^{-H}\} \right], \quad (4)$$

with $T_{YL} = (1 + aY^2 q_L^2 \xi^2)$, $T_L = (1 + a q_L^2 \xi^2)$. Equation (4) will be used for the calculation of $P(Y)$ or $P(\lambda)$ in combination with Eq. (2). For $H=0$ and $H=1$ we obtain from Eq. (4) the limiting forms, if we consider the identity $\ln(T) = \lim_{u \rightarrow 0} (1/u)(T^u - 1)$

$$G(Y)_{H=0} = \frac{1}{16} \left[\frac{E}{(1-\nu^2)\sigma_o} \right]^2 \frac{w^2}{a^2 \xi^2} \times \left[a q_L^2 \xi^2 (Y^2 - 1) + \ln \left(\frac{T_{YL}}{T_L} \right) \right] \quad (5)$$

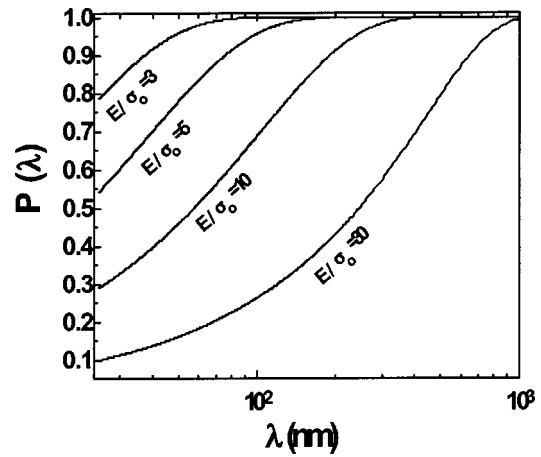


FIG. 4. $P(\lambda)$ vs lateral length scale λ for various applied loads σ_o/E , roughness correlation length $\xi = 200 \text{ nm}$, and roughness exponent $H = 0.3$.

$$G(Y)_{H=1} = \frac{1}{16} \left[\frac{E}{(1-\nu^2)\sigma_o} \right]^2 \frac{w^2}{a^2 \xi^2} \times \left[\ln \left(\frac{T_{YL}}{T_L} \right) - a(Y^2 - 1) q_L^2 \xi^2 (T_{YL} T_L)^{-1} \right]. \quad (6)$$

Furthermore our calculations are performed for macroscopic contact area of size $L = 100 \mu\text{m}$, $a_o = 0.3 \text{ nm}$, Poisson ratio $\nu = 0.3$, and rms roughness amplitude $w = 10 \text{ nm}$ such that $w \leq \xi$ (assuming nanometer scale roughness). Our investigations span a wide range of magnification values Y such that the corresponding length scales $\lambda (= L/Y)$ range from macroscopic dimensions ($\lambda \gg \xi$) down to length scales smaller than the corresponding roughness correlation length ξ .

Figure 1 shows calculations of the ratio $P(\lambda) = A(\lambda)/A(L)$ versus the lateral length scale λ for various roughness correlation lengths $\xi (\geq w)$. The function $P(\lambda)$ is obtained from $P(Y)$ if we substitute $Y = L/\lambda$. Clearly $P(\lambda)$ and thus the real contact area $A(\lambda)$ increases with lateral length scale λ and approaches values close to the macroscopic area $A(L)$ for $\lambda \gg \xi$. As the correlation length ξ decreases, i.e., the surface becomes rougher because the rms amplitude w is assumed fixed, the increment of $P(\lambda)$ and thus of the contact area becomes sharper for $\lambda > \xi$. The latter becomes more pronounced for roughness ratios $w/\xi > 0.1$ characterizing a rather rough surface at long wavelengths ($> \xi$).

On the other hand, if we plot $P(\lambda)$ as a function of the roughness correlation length ξ [Fig. 2(a)], we observe that as ξ increases and thus the surface smoothens at long wavelengths, the real contact area initially decreases and after passing through a minimum (for $\xi < \lambda$) it further increases. However, as Fig. 2(a) indicates, such an increment takes place at a faster rate for correlation lengths $\xi > \lambda$, and larger roughness exponents H , i.e., for smoother surfaces at short roughness wavelengths ($< \xi$). The minimum in Fig. 2(a) is due to the maximum of the factor G [Eq. (4)] which will lead to the inverse behavior for $P(\lambda)$ due to the Gaussian factor

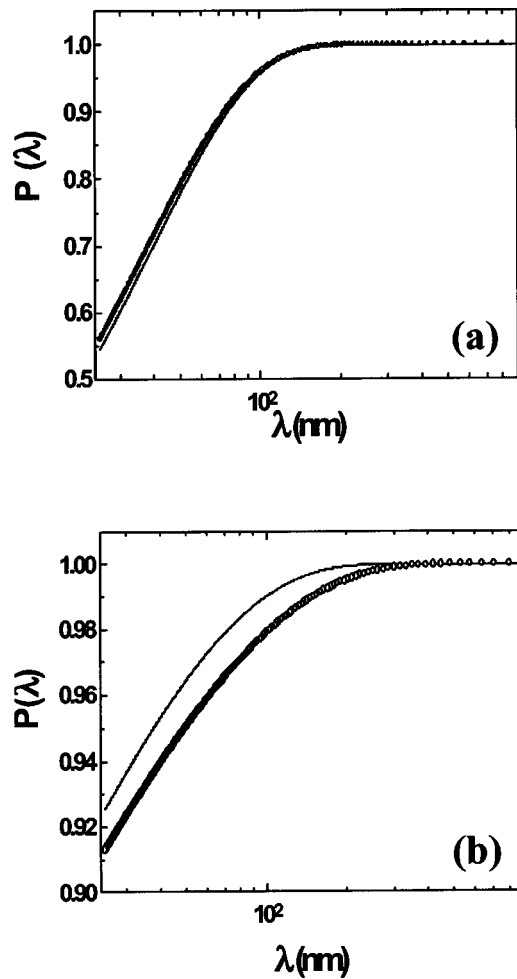


FIG. 5. $P(\lambda)$ vs lateral length scale λ for relatively high applied stress σ_o such that $E/\sigma_o=5$, and correlation length $\xi=200$ nm (a) $H=0.3$, and (b) $H=0.9$ (solid line) exact calculation based on Eq. (5) (circles) approximate calculation based on Eq. (9).

in the integrand of Eq. (2). Physically, as the correlation length ξ increases the ratio λ/ξ (for fixed contact length λ) decreases which favors lowering of the contact area, while an increment of the correlation length (for fixed roughness amplitude w) leads to surface smoothening which favors larger contact area. The competition between these processes lead to the minimum observed in Fig. 2(a).

Next, we investigate the dependence of $P(\lambda)$ on the applied load σ_o at some fixed lateral length scale λ comparable with the lateral roughness correlation length ξ . As Fig. 3

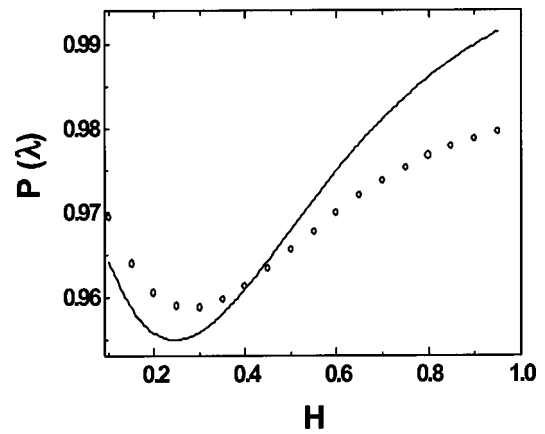


FIG. 6. $P(\lambda)$ vs roughness exponent H for lateral length scale $\lambda=100$ nm, $\xi=100$ nm, and relatively high applied stress σ_o such that $E/\sigma_o=5$: (solid line) exact calculation based on Eq. (5); (circles) approximate calculation based on Eq. (9).

indicates for small loads $\sigma_o/E < 0.2$, the dependence of $P(\lambda)$ on the ratio σ_o/E is linear [also see the inset of Fig. 3(a)], which is in agreement with former findings for weak loads.^{4,5} However, Fig. 3 indicates that the roughness effect either as a function of the roughness exponent H or the correlation length ξ is more pronounced in the nonlinear or relatively high load regime such that $0.2 < \sigma_o/E < 1$. Indeed, in this case rougher surfaces lead to lower values of the ratio $P(\lambda)$ and thus of the real contact area when λ is sufficiently smaller than ξ [see also Fig. 2(a)]. The opposite behavior takes place for small correlation lengths $\xi < \lambda/4$ [Fig. 2(a)]. Nevertheless, as the load approaches high values close to the elastic modulus, $\sigma_o \sim E$, the effect of roughness becomes insignificant. This is also depicted in Fig. 4 which shows that the ratio $P(\lambda)$ increases faster with increasing applied load σ_o or equivalently the real contact area approaches faster values close to the nominal macroscopic value $A(L)$.

Now we will compare our calculations with the approximations used in earlier works^{4,5} where the surface roughness spectrum is considered only by its asymptotic limits [following Eq. (3)]

$$C(q) = \begin{cases} w^2 \xi^2 / 2\pi & \text{if } q \ll 1/\sqrt{a}\xi \\ w^2 / (2\pi a^{1+H} \xi^{2H} q^{2(1+H)}) & \text{if } q \gg 1/\sqrt{a}\xi \end{cases} \quad (7)$$

with the wave vector $\tilde{q} = 1/\sqrt{a}\xi$ defined by the intersection of the saturation value of $C(q)$ and its power law regime. In such a case we obtain for the factor $G(Y)$ from Eq. (7)

$$G(Y) \approx \frac{1}{8} \left[\frac{E}{(1-v^2)\sigma_o} \right]^2 \begin{cases} \frac{w^2}{2a^{(1+H)}\xi^{2H}} \left\{ \frac{1}{1-H} \left[\left(\frac{2\pi Y}{L} \right)^{2(1-H)} - \left(\frac{1}{\xi\sqrt{a}} \right)^{2(1-H)} \right] \right\} \\ + \frac{w^2 \xi^2}{4} \left[\left(\frac{1}{\xi\sqrt{a}} \right)^4 - \left(\frac{2\pi}{L} \right)^4 \right] & \text{if } Y > Y_o \\ \frac{w^2}{2a^{(1+H)}\xi^{2H}} \left\{ \frac{1}{1-H} \left[\left(\frac{2\pi Y}{L} \right)^{2(1-H)} - \left(\frac{2\pi}{L} \right)^{2(1-H)} \right] \right\} & \text{if } Y < Y_o \end{cases} \quad (8)$$

with $Y_o = L/2\pi\xi\sqrt{a}$. Calculation of $P(Y)$ or $P(\lambda)$ versus lateral length scale λ (Fig. 5) indicate that such an approximation gives significantly smaller values for $P(\lambda)$ and thus for the real contact area $A(\lambda)$ especially for large roughness exponents $H > 0.5$ (see also Fig. 6). The estimated deviations for $P(\lambda)$ occur at length scales λ comparable to or smaller than the roughness correlation length ξ (Fig. 5).

Finally, we point out that for weak loads such that $\sigma_o \ll E$ and $G(Y) \gg 1$, it has been shown to lowest order in the expansion of $\sin x \approx x$ in Eq. (3) that $P(Y) \approx [\pi G(Y)]^{-1/2}$.⁵ However, for higher order terms, since $\sin x = \sum_{n=0, \infty} (-1)^n x^{2n+1}/(2n+1)!$ ($x < 1$), we obtain from Eqs. (3) and (4) the more precise expression and still in analytic form

$$A(\lambda) = A(L) \frac{1}{\sqrt{\pi}} \sum_{n=0}^{+\infty} 2^{2n+2} \frac{(-1)^n a^{2n+1}}{n!(2n+1)} \times \left(\frac{(1-\nu^2)\theta_0}{E} \right)^{2n+1} \left(\frac{\xi}{w} \right)^{2n+1} \left[\frac{1}{1-H} \{T_\lambda^{1-H} - T_L^{1-H}\} + \frac{1}{H} \{T_\lambda^{-H} - T_L^{-H}\} \right]^{-[n+(1/2)]}, \quad (9)$$

which shows that $A(\lambda)$ depends inversely proportionally to the long wavelength roughness ratio w/ξ in powers of $2n+1$, and proportionally to the stress ratio σ_o/E in powers of $2n+1$.

V. CONCLUSIONS

In summary, we have calculated the real contact area between elastic bodies with self-affine rough surfaces, which are described in terms of analytic correlation models in Fourier space. It is found that the roughness has a strong influence on the real contact area A at lateral length scales λ which are comparable with the in-plane roughness correlation length ξ , and for significant applied loads σ_o beyond the linear regime (or $A \propto \sigma_o$). The effect of the roughness exponent H can be rather complex, depending on the relative magnitude of the roughness correlation length ξ with respect to the lateral length scale λ where the contact area is calculated. We also show that descriptions of the influence of the roughness only based on power law approximations of the self-affine roughness spectrum can be rather inadequate, especially for large roughness exponents $H(>0.5)$.

Finally, we should point out that the calculation of the roughness influence is performed in terms of a specific roughness model¹¹ which gives an analytic form of the factor $G(Y)$ by incorporating the effect of intermediate lateral roughness wavelengths $q \sim 2\pi/\xi$. Clearly for other correlation models^{10,12} there can be deviations, since they differ mainly around the lateral roughness wavelengths $q \sim 2\pi/\xi$, however, not as strong as those that are obtained by use of extrapolation schemes as those of Eq. (8).

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